

# 3-7

## Equations of Lines in the Coordinate Plane

**Common Core State Standards**

**Prepares for G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines . . .

**MP 1, MP 3, MP 4**

**Objective** To graph and write linear equations



Think back!  
What did you learn in algebra that relates to steepness?



**Getting Ready!**

Ski resorts often use steepness to rate the difficulty of their hills. The steeper the hill, the higher the difficulty rating. Below are sketches of three new hills at a particular resort. Use each rating level only once. Which hill gets which rating? Explain.

**Difficulty Ratings**

- Easiest
- Intermediate
- ◆ Difficult

A

1190 ft

3300 ft

B

1180 ft

3000 ft

C

1150 ft

3500 ft



### Lesson Vocabulary

- slope
- slope-intercept form
- point-slope form

The Solve It involves using vertical and horizontal distances to determine steepness. The steepest hill has the greatest *slope*. In this lesson you will explore the concept of slope and how it relates to both the graph and the equation of a line.

**Essential Understanding** You can graph a line and write its equation when you know certain facts about the line, such as its slope and a point on the line.

Take note

### Key Concept Slope

#### Definition

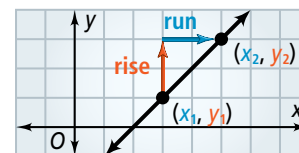
The **slope**  $m$  of a line is the ratio of the vertical change (**rise**) to the horizontal change (**run**) between any two points.

#### Symbols

A line contains the points  $(x_1, y_1)$  and  $(x_2, y_2)$ .

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

#### Diagram



## Plan

How do you know which numbers go where in a formula?

For slope, you can choose either point's  $y$ -coordinate as  $y_2$ . Just be sure to use the same point's  $x$ -coordinate as  $x_2$ .



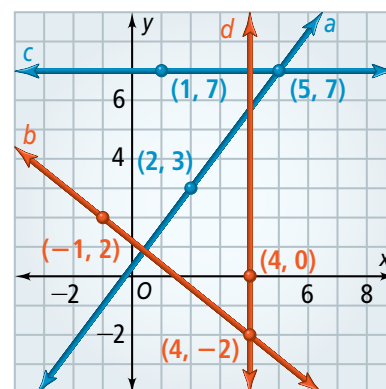
### Problem 1 Finding Slopes of Lines

**A** What is the slope of line  $b$ ?

$$\begin{aligned} m &= \frac{2 - (-2)}{-1 - 4} \\ &= \frac{4}{-5} \\ &= -\frac{4}{5} \end{aligned}$$

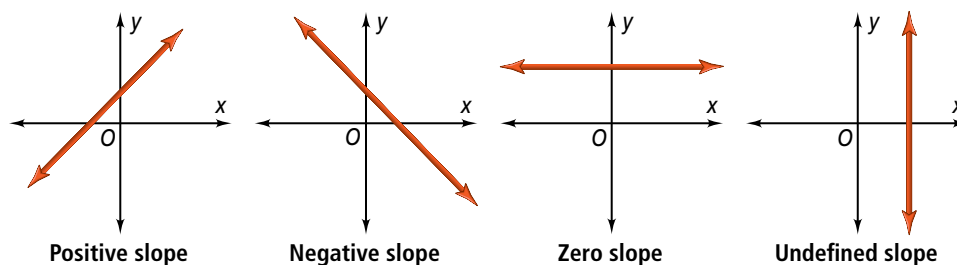
**B** What is the slope of line  $d$ ?

$$\begin{aligned} m &= \frac{0 - (-2)}{4 - 4} \\ &= \frac{2}{0} \text{ Undefined} \end{aligned}$$



- Got It?** 1. Use the graph in Problem 1.
- What is the slope of line  $a$ ?
  - What is the slope of line  $c$ ?

As you saw in Problem 1 and Got It 1 the slope of a line can be positive, negative, zero, or undefined. The sign of the slope tells you whether the line rises or falls to the right. A slope of zero tells you that the line is horizontal. An undefined slope tells you that the line is vertical.



You can graph a line when you know its equation. The equation of a line has different forms. Two forms are shown below. Recall that the  $y$ -intercept of a line is the  $y$ -coordinate of the point where the line crosses the  $y$ -axis.

Take note

### Key Concept Forms of Linear Equations

#### Definition

The **slope-intercept form** of an equation of a nonvertical line is  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept.

The **point-slope form** of an equation of a nonvertical line is  $y - y_1 = m(x - x_1)$ , where  $m$  is the slope and  $(x_1, y_1)$  is a point on the line.

#### Symbols

$$y = \underset{\substack{\uparrow \\ \text{slope}}}{m}x + \underset{\substack{\uparrow \\ \text{y-intercept}}}{b}$$

$$\underset{\substack{\uparrow \\ \text{y-coordinate}}}{y} - \underset{\substack{\uparrow \\ \text{y-coordinate}}}{y_1} = \underset{\substack{\uparrow \\ \text{slope}}}{m}(x - \underset{\substack{\uparrow \\ \text{x-coordinate}}}{x_1})$$

## Plan

### What do you do first?

Determine which form of linear equation you have. Then use the equation to identify the slope and a starting point.

## Think

### What do you do when the slope is not a fraction?

A rise always needs a run. So, write the integer as a fraction with 1 as the denominator.



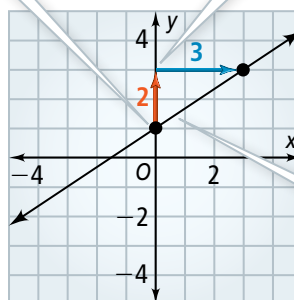
## Problem 2 Graphing Lines

**A** What is the graph of  $y = \frac{2}{3}x + 1$ ?

The equation is in slope-intercept form,  $y = mx + b$ . The slope  $m$  is  $\frac{2}{3}$  and the  $y$ -intercept  $b$  is 1.

**Step 1** Graph a point at  $(0, 1)$ .

**Step 2** Use the slope  $\frac{2}{3}$ . Go up 2 units and right 3 units. Graph a point.



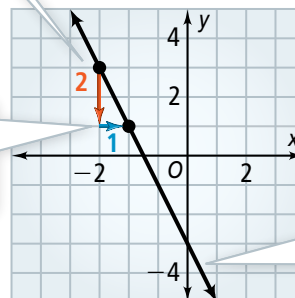
**Step 3** Draw a line through the two points.

**B** What is the graph of  $y - 3 = -2(x + 3)$ ?

The equation is in point-slope form,  $y - y_1 = m(x - x_1)$ . The slope  $m$  is  $-2$  and a point  $(x_1, y_1)$  on the line is  $(-3, 3)$ .

**Step 1** Graph a point at  $(-3, 3)$ .

**Step 2** Use the slope  $-2$ . Go down 2 units and right 1 unit. Graph a point.



**Step 3** Draw a line through the two points.



**Got It?** 2. a. Graph  $y = 3x - 4$ .  
b. Graph  $y - 2 = -\frac{1}{3}(x - 4)$ .

You can write an equation of a line when you know its slope and at least one point on the line.

## Plan

Which linear equation form should you use?

When you know the slope and the  $y$ -intercept, use slope-intercept form.  
When you know the slope and a point on the line, use point-slope form.



### Problem 3 Writing Equations of Lines

**A** What is an equation of the line with slope 3 and  $y$ -intercept  $-5$ ?

$$y = mx + b$$

$$m = 3$$

$$b = -5$$

$$y = 3x + (-5) \quad \text{Substitute 3 for } m \text{ and } -5 \text{ for } b.$$

$$y = 3x - 5 \quad \text{Simplify.}$$

**B** What is an equation of the line through  $(-1, 5)$  with slope 2?

$$y - y_1 = m(x - x_1)$$

$$y_1 = 5$$

$$m = 2$$

$$x_1 = -1$$

$$y - 5 = 2[x - (-1)] \quad \text{Substitute } (-1, 5) \text{ for } (x_1, y_1) \text{ and } 2 \text{ for } m.$$

$$y - 5 = 2(x + 1) \quad \text{Simplify.}$$



**Got It?** 3. a. What is an equation of the line with slope  $-\frac{1}{2}$  and  $y$ -intercept 2?  
b. What is an equation of the line through  $(-1, 4)$  with slope  $-3$ ?

Postulate 1-1 states that through any two points, there is exactly one line. So, you need only two points to write the equation of a line.



### Problem 4 Using Two Points to Write an Equation

What is an equation of the line at the right?

## Plan

What is the first thing you need to know?

It doesn't matter yet what form of linear equation you plan to use. You'll need the slope for *both* slope-intercept form and point-slope form.

## Think

Start by finding the slope  $m$  of the line through the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{3 - (-2)} = \frac{6}{5}$$

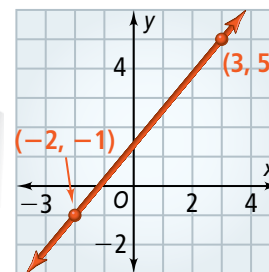
You have the slope and you know two points on the line. Use point-slope form.

$$y - y_1 = m(x - x_1)$$

Use either point for  $(x_1, y_1)$ . For example, you can use  $(3, 5)$ .

$$y - 5 = \frac{6}{5}(x - 3)$$

## Write





- Got It?** 4. a. What is the equation of the line in Problem 4 if you use  $(-2, -1)$  instead of  $(3, 5)$  in the last step?  
b. Rewrite the equations in Problem 4 and part (a) in slope-intercept form and compare them. What can you conclude?

You know that the slope of a horizontal line is 0 and the slope of a vertical line is undefined. Thus, horizontal and vertical lines have easily recognized equations.

## Think

How is this different from writing other linear equations?

You don't need the slope. Just locate the point where the line crosses the  $x$ -axis (for vertical) or  $y$ -axis (for horizontal).

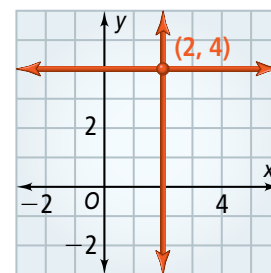


### Problem 5 Writing Equations of Horizontal and Vertical Lines

What are the equations for the horizontal and vertical lines through  $(2, 4)$ ?

Every point on the horizontal line through  $(2, 4)$  has a  $y$ -coordinate of 4. The equation of the line is  $y = 4$ . It crosses the  $y$ -axis at  $(0, 4)$ .

Every point on the vertical line through  $(2, 4)$  has an  $x$ -coordinate of 2. The equation of the line is  $x = 2$ . It crosses the  $x$ -axis at  $(2, 0)$ .



- Got It?** 5. a. What are the equations for the horizontal and vertical lines through  $(4, -3)$ ?  
b. **Reasoning** Can you write the equation of a vertical line in slope-intercept form? Explain.



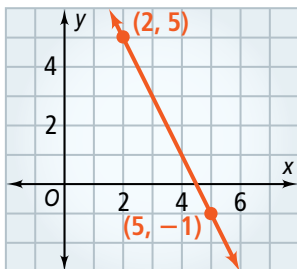
## Lesson Check

### Do you know HOW?

For Exercises 1 and 2, find the slope of the line passing through the given points.

1.  $(4, 5)$  and  $(6, 15)$

2.



3. What is an equation of a line with slope 8 and  $y$ -intercept 10?  
4. What is an equation of a line passing through  $(3, 3)$  and  $(4, 7)$ ?

### Do you UNDERSTAND?



MATHEMATICAL PRACTICES

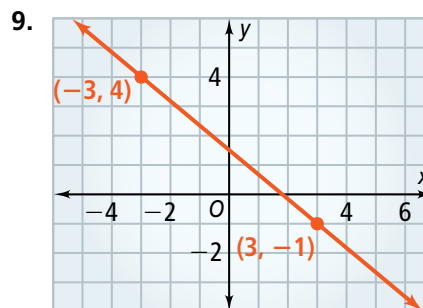
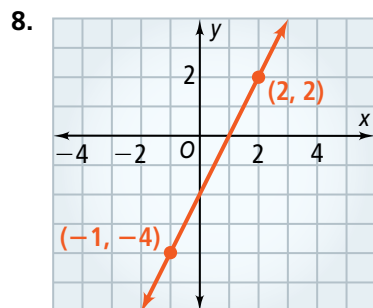
5. **Vocabulary** Explain why you think *slope-intercept form* makes sense as a name for  $y = mx + b$ . Explain why you think *point-slope form* make sense as a name for  $y - y_1 = m(x - x_1)$ .  
6. **Compare and Contrast** Graph  $y = 2x + 5$  and  $y = -\frac{1}{3}x + 5$ . Describe how these lines are alike and how they are different.  
7. **Error Analysis** A classmate found the slope of the line passing through  $(8, -2)$  and  $(8, 10)$ , as shown at the right. Describe your classmate's error. Then find the correct slope of the line passing through the given points.

$$\begin{aligned} m &= \frac{8 - 8}{10 - (-2)} \\ m &= \frac{0}{12} \\ m &= 0 \end{aligned}$$

## A Practice

Find the slope of the line passing through the given points.

See Problem 1.



10.  $(4, -6), (7, 2)$

11.  $(-3, 7), (-1, 4)$

12.  $(-8, 3), (-11, 4)$

13.  $(-6, 2), (-7, 10)$

14.  $(3, 2), (-6, 2)$

15.  $(5, 9), (5, -6)$

Graph each line.

See Problem 2.

16.  $y = x + 2$

17.  $y = 3x + 4$

18.  $y = \frac{1}{2}x - 1$

19.  $y = -\frac{5}{3}x + 2$

20.  $y - 3 = \frac{1}{3}(x - 3)$

21.  $y - 1 = -3(x + 2)$

22.  $y + 4 = (x - 5)$

23.  $y + 1 = -\frac{2}{3}(x + 4)$

Use the given information to write an equation of each line.

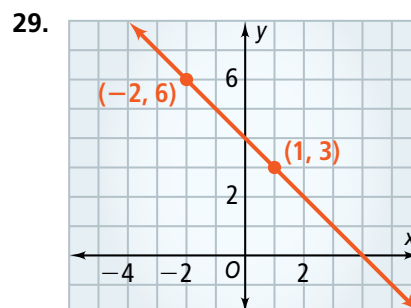
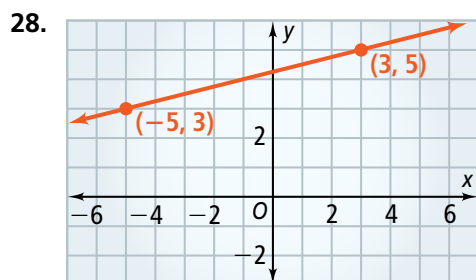
See Problems 3 and 4.

24. slope 3, y-intercept 6

25. slope  $\frac{1}{2}$ , y-intercept  $-5$

26. slope  $\frac{2}{3}$ , passes through  $(-2, -6)$

27. slope  $-3$ , passes through  $(4, -1)$



30. passes through  $(0, 5)$  and  $(5, 8)$

31. passes through  $(6, 2)$  and  $(2, 4)$

32. passes through  $(-4, 4)$  and  $(2, 10)$

33. passes through  $(-1, 0)$  and  $(-3, -1)$

Write the equation of the horizontal and vertical lines through the given point.

See Problem 5.

34.  $(4, 7)$

35.  $(3, -2)$

36.  $(0, -1)$

37.  $(6, 4)$

**B Apply**

Graph each line.

38.  $x = 3$

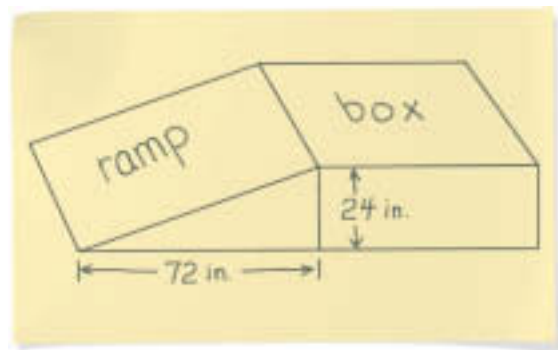
39.  $y = -2$

40.  $x = 9$

41.  $y = 4$

**© 42. Open-Ended** Write equations for three lines that contain the point (5, 6).**© 43. Think About a Plan** You want to construct a “funbox” at a local skate park. The skate park’s safety regulations allow for the ramp on the funbox to have a maximum slope of  $\frac{4}{11}$ . If you use the funbox plan at the right, can you build the ramp to meet the safety regulations? Explain.

- What information do you have that you can use to find the slope?
- How can you compare slopes?



Write each equation in slope-intercept form.

44.  $y - 5 = 2(x + 2)$

45.  $y + 2 = -(x - 4)$

46.  $-5x + y = 2$

47.  $3x + 2y = 10$

**STEM 48. Science** The equation  $P = -\frac{1}{33}d + 1$  represents the pressure  $P$  in atmospheres a scuba diver feels  $d$  feet below the surface of the water.

- What is the slope of the line?
- What does the slope represent in this situation?
- What is the  $y$ -intercept ( $P$ -intercept)?
- What does the  $y$ -intercept represent in this situation?

Graph each pair of lines. Then find their point of intersection.

49.  $y = -4, x = 6$

50.  $x = 0, y = 0$

51.  $x = -1, y = 3$

52.  $y = 5, x = 4$

**STEM 53. Accessibility** By law, the maximum slope of an access ramp in new construction is  $\frac{1}{12}$ . The plan for the new library shows a 3-ft height from the ground to the main entrance. The distance from the sidewalk to the building is 10 ft. If you assume the ramp does not have any turns, can you design a ramp that complies with the law? Explain.

- What is the slope of the  $x$ -axis? Explain.
- Write an equation for the  $x$ -axis.
- What is the slope of the  $y$ -axis? Explain.
- Write an equation for the  $y$ -axis.

**© 56. Reasoning** The  $x$ -intercept of a line is 2 and the  $y$ -intercept is 4. Use this information to write an equation for the line.**57. Coordinate Geometry** The vertices of a triangle are  $A(0, 0)$ ,  $B(2, 5)$ , and  $C(4, 0)$ .

- Write an equation for the line through  $A$  and  $B$ .
- Write an equation for the line through  $B$  and  $C$ .
- Compare the slopes and the  $y$ -intercepts of the two lines.

**Challenge**

Do the three points lie on one line? Justify your answer.

58.  $(5, 6), (3, 2), (6, 8)$

59.  $(-2, -2), (4, -4), (0, 0)$

60.  $(5, -4), (2, 3), (-1, 10)$

Find the value of  $a$  such that the graph of the equation has the given slope.

61.  $y = \frac{2}{9}ax + 6; m = 2$

62.  $y = -3ax - 4; m = \frac{1}{2}$

63.  $y = -4ax - 10; m = -\frac{2}{3}$

**Standardized Test Prep**

SAT/ACT

64.  $\overline{AB}$  has endpoints  $A(8, k)$  and  $B(7, -3)$ . The slope of  $\overline{AB}$  is 5. What is  $k$ ?

(A) 1

(B) 2

(C) 5

(D) 8

65. Two angles of a triangle measure 68 and 54. What is the measure of the third angle?

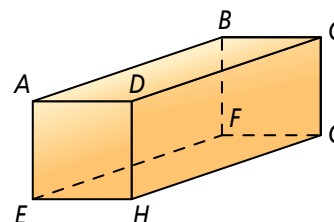
(F) 14

(G) 58

(H) 122

(I) 180

66. Which of the following CANNOT be true?

(A) plane  $ABCD \parallel$  plane  $EFGH$ (B) Planes  $ABCD$  and  $CDHG$  intersect in  $\overleftrightarrow{CD}$ .(C)  $ABCD$  and  $ABC$  represent the same plane.(D) plane  $ADHE \parallel$  plane  $DCG$ 67. The length of a rectangle is  $(x - 2)$  inches and the width is  $5x$  inches. Which expression represents the perimeter of the rectangle in inches?(F)  $6x - 2$ (G)  $12x - 4$ (H)  $5x^2 - 10x$ (I)  $10x^2 - 20x$ 

Short Response

68. One of the angles in a certain linear pair is acute. Your friend says the other angle must be obtuse. Is your friend's conjecture reasonable? Explain.

**Mixed Review**

For Exercises 69 and 70, construct the geometric figure.

69. a rectangle with a length twice its width

70. a square

See Lesson 3-6.

Name the property that justifies each statement.

See Lesson 2-5.

71.  $4(2a - 3) = 8a - 12$

72. If  $b + c = 7$  and  $b = 2$ , then  $2 + c = 7$ .

73.  $\overline{RS} \cong \overline{RS}$

74. If  $\angle 1 \cong \angle 4$ , then  $\angle 4 \cong \angle 1$ .

**Get Ready!** To prepare for Lesson 3-8, do Exercises 75–77.

Find the slope of the line passing through the given points.

See Lesson 3-7.

75.  $(2, 5), (-2, 3)$

76.  $(0, -5), (2, 0)$

77.  $(1, 1), (2, -4)$